Harmonic generation in the beam current in a traveling wave tube

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Introduction

- Charge overtaking, or orbital crowding, can lead to harmonic generation, even in linear regime.

- This is well-known in klystron, but never studied in traveling wave tube (TWT).

- This paper extends the klystron theory of orbital bunching to a TWT to compute the harmonic content in the beam current [1].

[1] C. F. Dong, *et al*., *IEEE Trans. Electron Devices* (accepted, 2015).

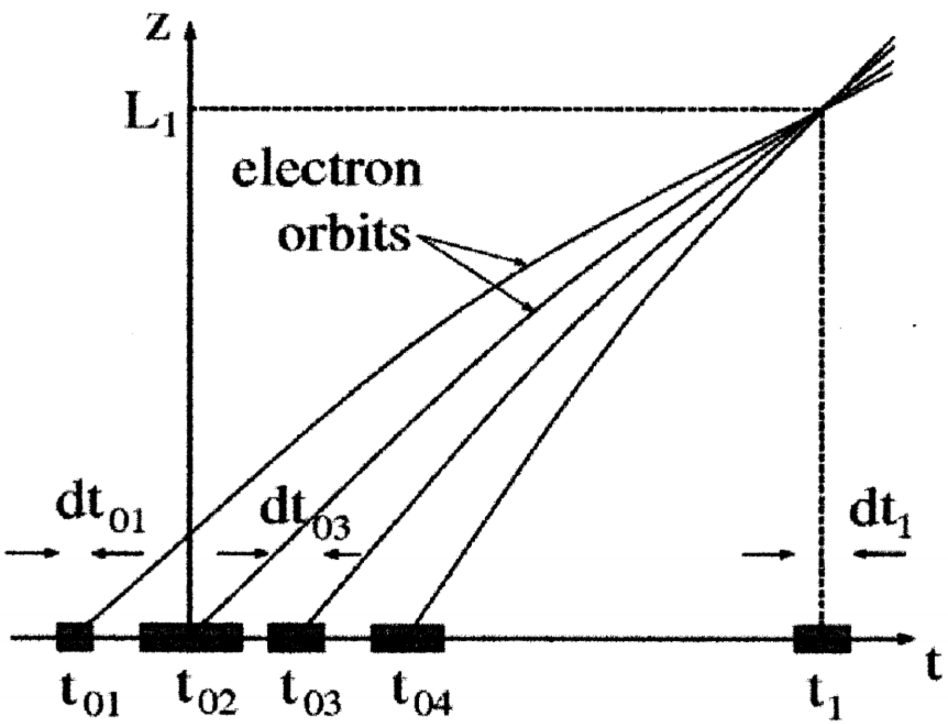
Harmonic generation due to orbital crowding

Unperturbed orbit:

$$z = z_0(t, t_0) = v_0(t - t_0)$$

With input signal:

$$z = z_0(t, t_0) + z_1(t, t_0)$$



At $t = t_1$, $z = L_1$, current = infinite
=> significant harmonic content

Linear Theory of TWT

$$z_1(t,t_0) = \mathrm{Re}[Z_1(t,t_0)]$$

Pierce's 3-wave theory [2,3] :

$$\frac{d^3 Z_1}{dt^3} + j\omega_0 C(b - jd)\frac{d^2 Z_1}{dt^2} + 4QC^3\omega_0^2 \frac{dZ_1}{dt} + j\omega_0^3 C^3[4QC(b - jd) + (1 + Cb)^2]Z_1 = 0$$

C: gain parameter
b: detune
Q: "space charge effect"
d: cold-tube circuit loss

Initial conditions

$$Z_1(t = t_0) = 0,$$

$$\dot{Z}_1(t = t_0) = 0,$$

$$\ddot{Z}_1(t = t_0) = \frac{e}{m} E_{10} e^{j\omega_0 t_0}.$$

[2] I. M. Rittersdorf, T. M. Antonsen, Jr., D. Chernin, and Y. Y. Lau, IEEE J. Electron Device Soc. 1, 117 (2013).

[3] J. R. Pierce, *Traveling Wave Tubes*, Van Nostrand (New York, 1950).

Linear Theory of TWT (cont'd)

$$Z_1(t,t_0)=\frac{eE_{10}}{m\omega_0^2C^2}e^{j\omega_0 t_0}\left[\alpha_1 e^{C\omega_0\delta_1(t-t_0)}+\alpha_2 e^{C\omega_0\delta_2(t-t_0)}+\alpha_3 e^{C\omega_0\delta_3(t-t_0)}\right],$$

Pierce's 3-wave dispersion relation [3]:

$$(\delta^2+4QC)(\delta+jb+d)=-j(1+Cb)^2$$

α_1, α_2, and α_3 determined from (launching loss):

$$\alpha_1+\alpha_2+\alpha_3=0,$$
$$\alpha_1\delta_1+\alpha_2\delta_2+\alpha_3\delta_3=0,$$
$$\alpha_1\delta_1^2+\alpha_2\delta_2^2+\alpha_3\delta_3^2=1.$$

[3] J. R. Pierce, *Traveling Wave Tubes*, Van Nostrand (New York, 1950).

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TWT Bunching Parameter, X (new)

$$L = v_0(t - t_0) + z_1(t, t_0)$$

yields relationship between the departure time (t_0) and the arrival time (t_1)

$$\omega_0(t - t_0) = \frac{\omega_0 L}{v_0} - \frac{\omega_0 z_1(t, t_0)}{v_0} \equiv \frac{\omega_0 L}{v_0} - X \,\mathrm{Re}\left[e^{i\omega_0 t_0} R(t - t_0)\right]$$

$$R(t - t_0) = \alpha_1 e^{C\omega_0 \delta_1 (t - t_0)} + \alpha_2 e^{C\omega_0 \delta_2 (t - t_0)} + \alpha_3 e^{C\omega_0 \delta_3 (t - t_0)}.$$

$$X = \sqrt{\frac{2}{C}\left(\frac{P_{in}}{P_b}\right)}$$

P_{in} : input power

P_b : DC beam power

$v_w = eE_{10} / m\omega_0$

X= dimensionless “bunching parameter”

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Relation between arrival time (t_1) and departure time (t_0)

$$\omega_0 (t^{(0)} - t_0) = \frac{\omega_0 L}{v_0}$$

Zeroth order

$$\omega_0 (t^{(k)} - t_0) = \frac{\omega_0 L}{v_0} - X \,\mathrm{Re}\left[e^{i\omega_0 t_0} R(t^{(k-1)} - t_0) \right], \qquad k = 1, 2, 3, \ldots.$$

k-th order
(k = 4 suffices)

Harmonic Content of AC current on TWT [4]

$$I_L(t)dt = I_0 dt_0$$

charge conservation

$$I_L(t) = \sum_{n=-\infty}^{\infty} \tilde{I}_n e^{jn\omega_0 t}$$

contains n-th harmonic

$$\tilde{I}_n = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} dt I_L(t) e^{-in\omega_0 t} = \frac{\omega_0}{2\pi} I_0 \int_0^{2\pi/\omega_0} dt_0 e^{-in\omega_0 t} = \frac{I_0}{2\pi}\int_0^{2\pi} d(\omega_0 t_0) e^{-in\omega_0 t_0 - in\omega_0 (t-t_0)}$$

[4] C. B. Wilsen, Y. Y. Lau, D. P. Chernin, and R. M. Gilgenbach, *IEEE Trans. Plasma Sciences* **30**, 1176 (2002).

Small signal electric field (E_1) on Circuit

Linearized force law

$$\frac{d^2 Z_1}{dt^2} + \omega_0^2 4QC^3 Z_1 = \frac{e}{m} E_1(z,t)$$

$$E_1(0,t) = E_{10} e^{j\omega_0 t}$$

$$\Rightarrow \quad \left| \frac{E_1(z,t)}{E_1(0,t)} \right|^2 = \left| \sum_{i=1}^{3} \alpha_i \delta_i^2 e^{C\delta_i(\omega_0 z/v_0)} + 4QC \sum_{i=1}^{3} \alpha_i e^{C\delta_i(\omega_0 z/v_0)} \right|^2$$

This gives the rf power gain on circuit according to linear theory.

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Example: C-band TWT, P_{in} = 1mW

$\omega_0 = 2\pi \times 4.5 GHz,$

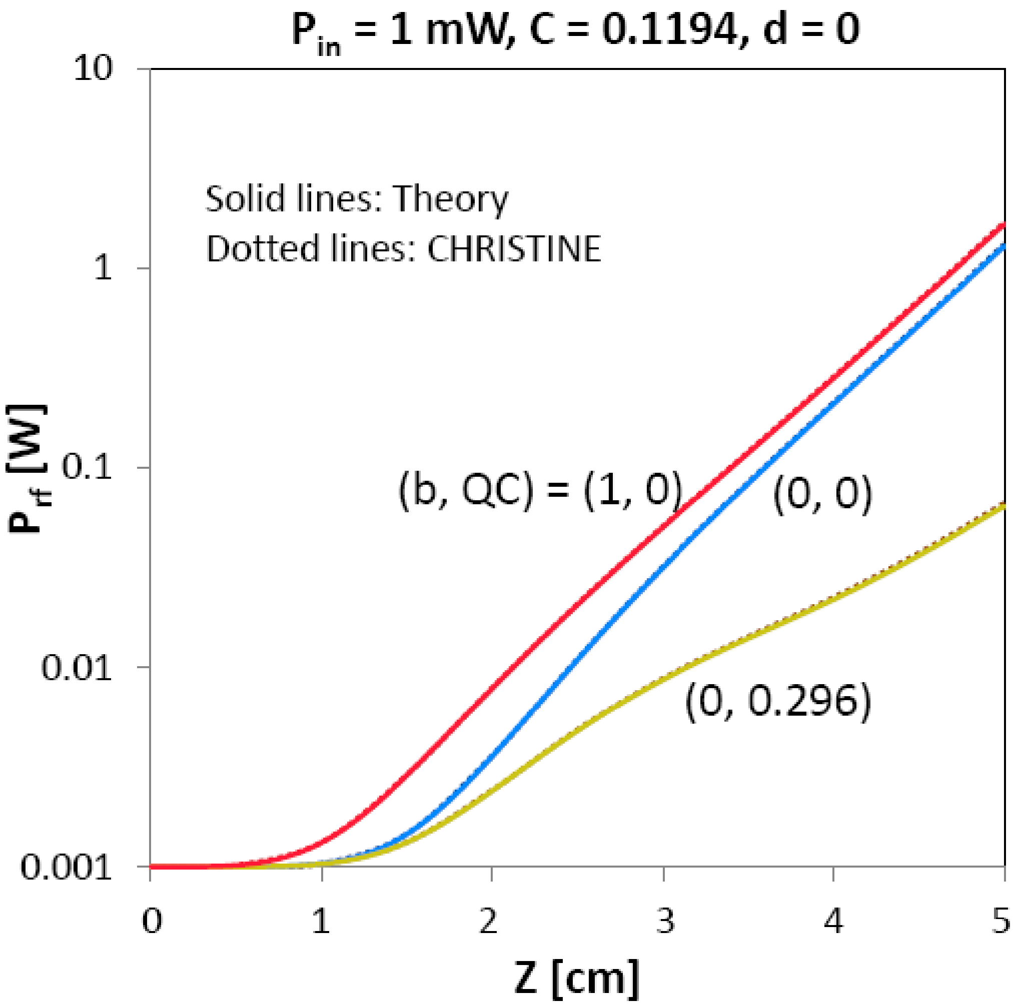
V_b = 2.776 kV,
I_0 = 0.17 A,
C = 0.1194,
K = 111.2 ohm,

$v_0 = 5.93 \times 10^7 m/s$

$P_b = V_b I_0$ = 417.9 W

$I_0 / V_b^{3/2} = 1.16$ micro-perveance

Excellent agreement for power gain on circuit, at low drive power



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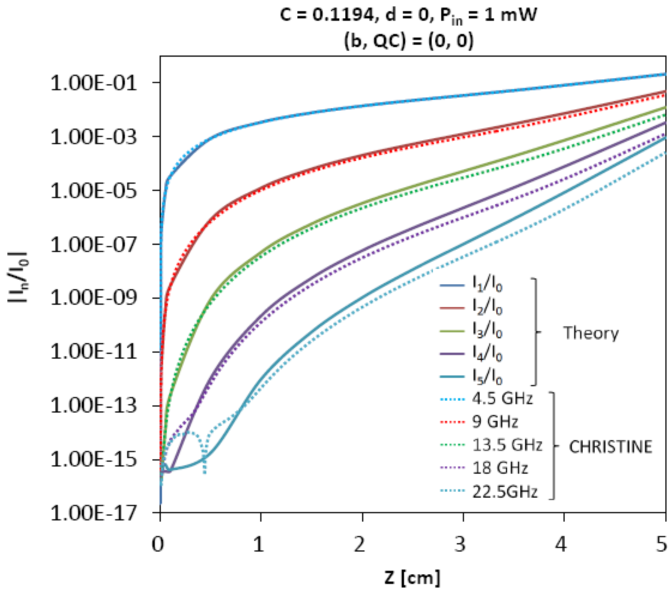
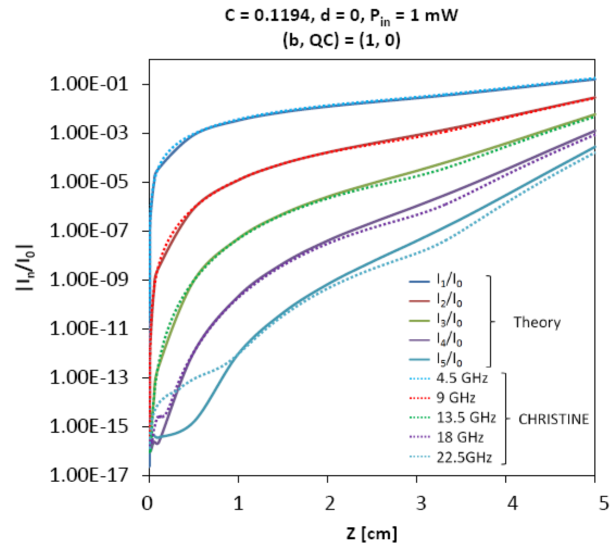
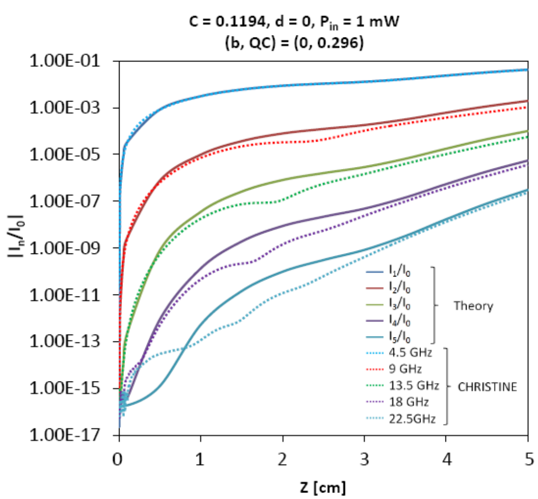




Excellent agreement for harmonic content, at low drive power



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Example: C-band TWT, P_{in} = 1mW

Harmonic content at z = 4cm for 1 mW drive

$$\omega_0 = 2\pi \times 4.5GHz,$$

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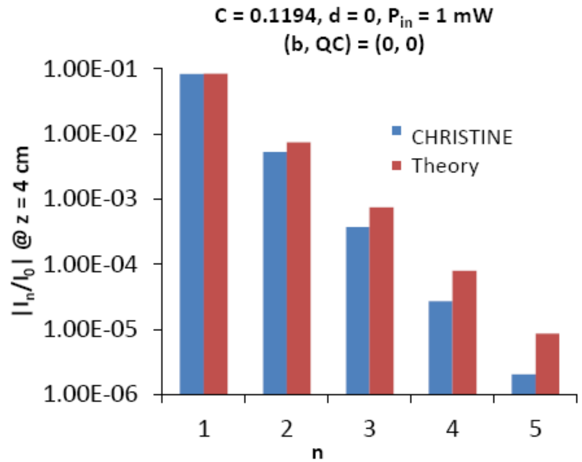

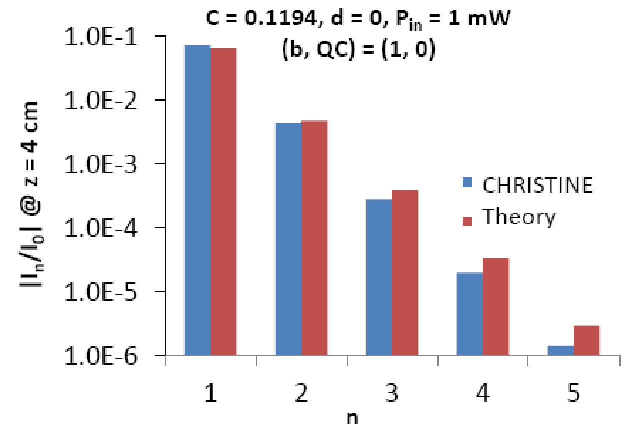

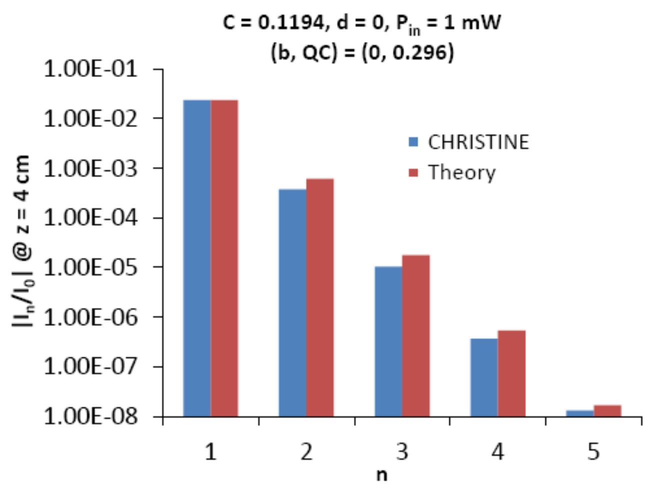

Excellent agreement
for harmonic content,
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Example: C-band TWT, P_{in} = 54mW

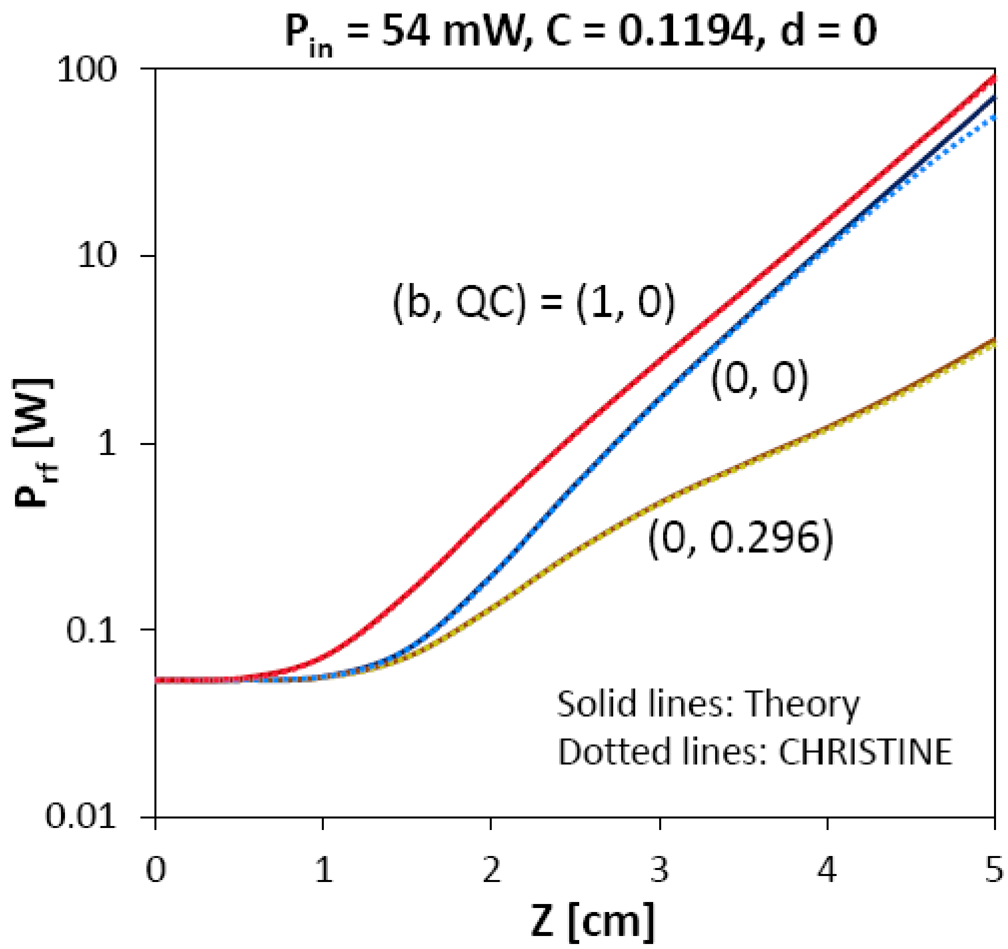
$$\omega_0 = 2\pi \times 4.5GHz,$$

V_b = 2.776 kV,
I_0 = 0.17 A,
C = 0.1194,
K = 111.2 ohm,

$$v_0 = 5.93 \times 10^7 \, m/s$$

$P_b = V_b I_0$ = 417.9 W

$I_0 / V_b^{3/2} = 1.16$ micro-perveance



Excellent agreement for power gain on circuit, at high drive power

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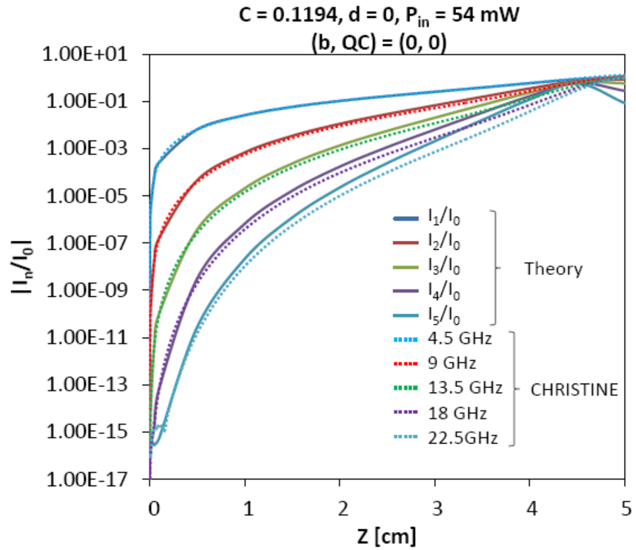

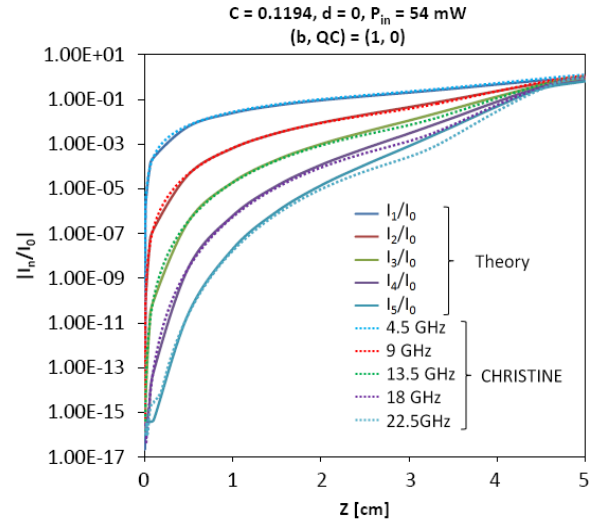

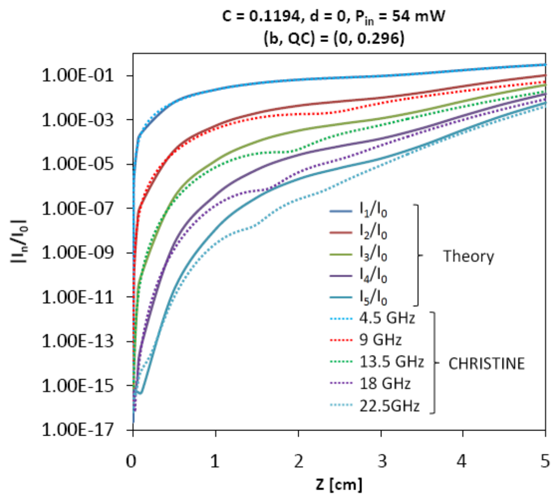

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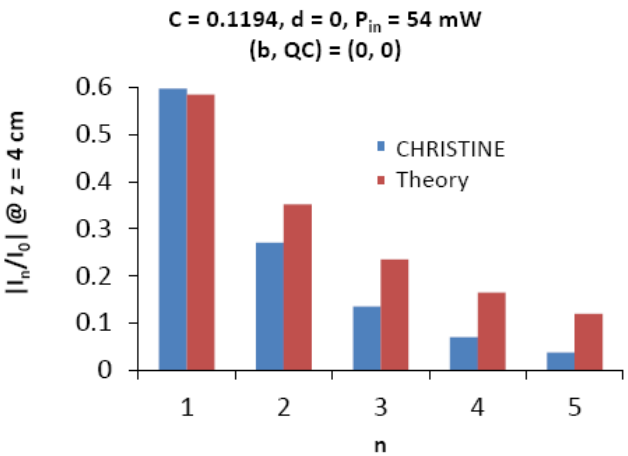
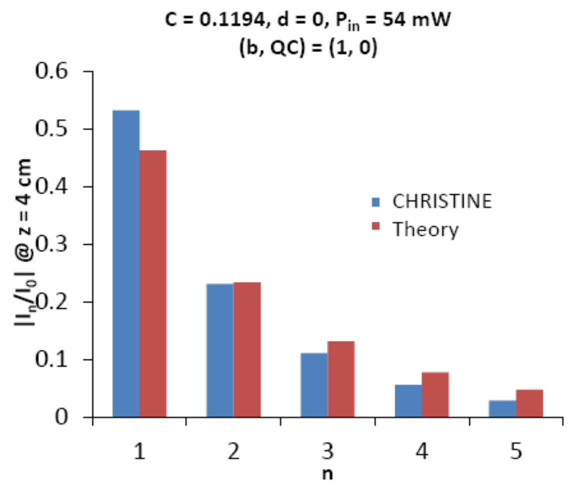
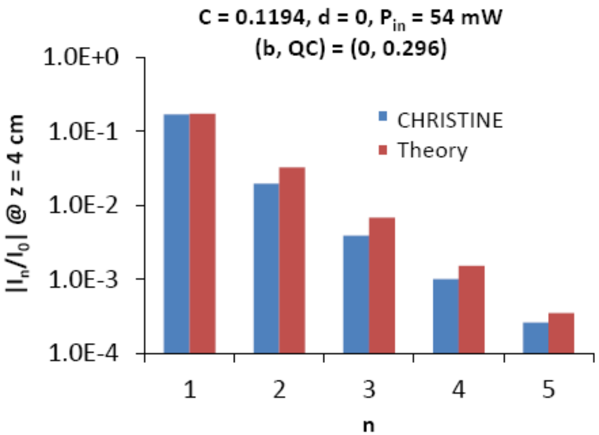
Harmonic content at z = 4cm for 54 mW drive







Excellent agreement for harmonic content, at high drive power

Conclusion

- Crowding of the linearized electron orbits leads to significant harmonic content in the AC current. This is purely a *kinematic* effect (charge conservation).
- The harmonic currents calculated analytically agree with CHRISTINE simulation results.
- These harmonic AC currents may be used for frequency multiplication or TWT linearization (future work).