Probabilistic evaluation of closure models for the Hall thruster anomalous collision frequency

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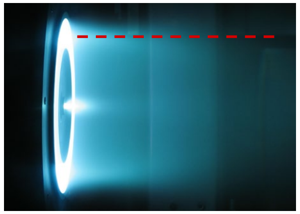
Problem: Current Hall thruster simulations are not predictive due to incomplete understanding of electron transport physics.

Hall thrusters

Annular $\vec{E} \times \vec{B}$ discharge used to accelerate ions for spacecraft propulsion

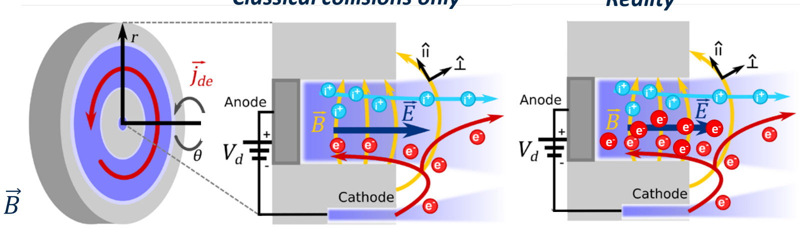


H9 Hall thruster, developed by Jet Propulsion Laboratory, Air Force Research Laboratory, and University of Michigan

Anomalous electron transport

Electrons diffuse across magnetic field lines much faster than classical theory predicts, so simulations cannot match experiment without hand-tuning. Model as extra "anomalous collision frequency" (ν_{AN}) in electron momentum equation

Ohm's law: $(\nu_e + \nu_{AN}) \frac{m_e}{q} \vec{j}_e = q n_e \vec{E} + \nabla P_e - \vec{j}_e \times \vec{B}$



Question: How can we develop and test models of anomalous transport while accounting for model uncertainty?

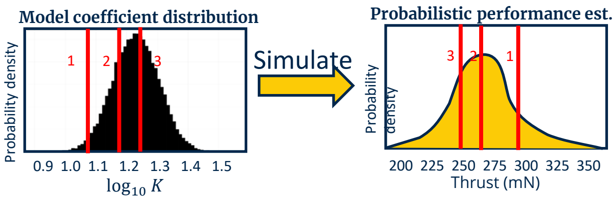
Approach: Use validated simulations as surrogate data to calibrate and test models of anomalous transport

Model (example):[1] $\nu_{AN} = \frac{1}{K} \frac{|\nabla \cdot (\mathbf{u_i} n_e T_e)|}{m_e c_s n_e v_{de}}$

Coefficients: $\{K\}$

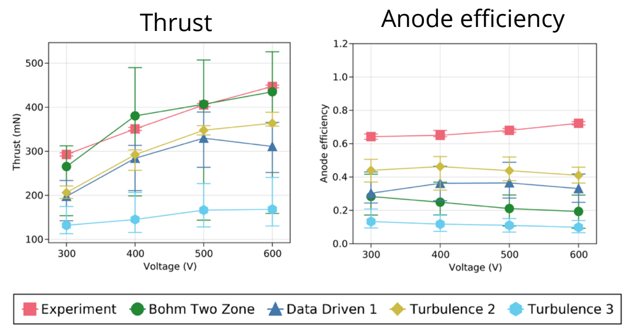
Surrogate data (hand-tuned profiles:

Obtain probability distribution of model coefficients, sample and run simulations of H9 Hall thruster (not in training dataset)



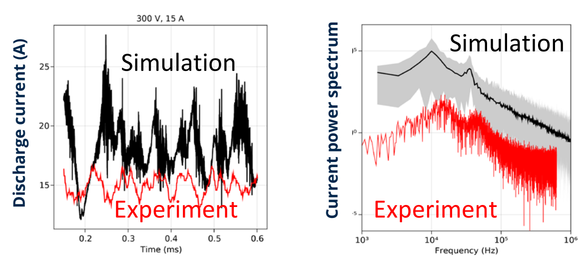
Model	Expression for ν_{AN}	Coefficients	Proposed mechanism
Two-zone Bohm	$\begin{cases} \alpha_0\, \omega_{ce} & z < L \\ \alpha_1\, \omega_{ce} & z \geq L \end{cases}$ [2]	$\alpha_0,\ \alpha_1$	Bohm diffusion
Turbulence II	$\frac{1}{K} \frac{\lvert\nabla \cdot (\mathbf{u_i} n_e T_e)\rvert}{m_e c_s n_e v_{de}}$ [1]	K	Azimuthal instability saturated by ion-wave trapping and wave convection
Turbulence III	$\frac{1}{K} \omega_{ce} \left(\frac{1}{1 + (C\ \nabla v_{de})^\alpha} \right)$ [4]	$K,\ C,\ \alpha$	Turbulent transport suppressed by shear stress
Data-driven	$\omega_{ce} \left(c_0 + \frac{c_1 \lvert\mathbf{u_i}\rvert}{c_2 c_s + v_{de}} \right)$ [3]	$c_0,\ c_1,\ c_2$	None

Application: Evaluate multiple algebraic models to determine if any are predictive and extensible



Data-driven model performs almost as well as best first-principles model

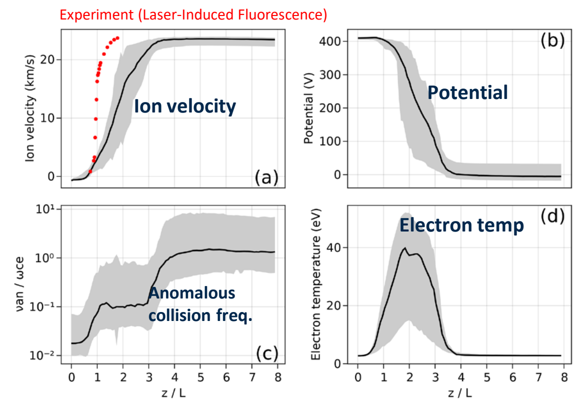
Breathing-like oscillations produced by all models (~ 10 kHz)



Simulations performed using Hall2De, 2D axisymmetric fluid code developed at JPL

Discussion: Examine reasons for low performance

Axial properties for data-driven model



Relaxed acceleration profile due to large anomalous transport near exit-plane yields low beam, divergence, mass utilization efficiencies. Algebraic models may not have sufficient feedback to steepen profiles.

Conclusion:

- Algebraic models tend to under-predict performance when calibrated on steady-state data
- Breathing-like oscillations consistently reproduced
- Bayesian techniques can quantify model uncertainty

Nomenclature:

- ν_{AN}: Anomalous collision freq.
- $\vec{B}$: Magnetic field
- $\vec{E}$: Electric field
- ν_e: Classical electron collision freq.
- ω_{ce}: Electron cyclotron freq.
- m_e: Electron mass
- n_e: Electron number density
- T_e: Electron temp.
- P_e: Electron pressure
- $\vec{j}_e$: Electron current density
- v_{de}: Electron drift speed
- $\mathbf{u_i}$: Ion velocity
- c_s: Ion acoustic speed

References:
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[2] B. Jorns, Plasma Sources Sci. Technol. 27 104007 (2018)
[3] K. Hara et al, J. Appl. Phys. 115, 203304 (2014)
[4] M. K. Scharfe et al, IEEE Transactions on Plasma Science 36, 05 (2008).

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